LINEUP WITH MATH™

Math-Based Decisions in Air Traffic Control
for Grades 5 - 9

Problem Set D

Understanding the Effects of Differences in Speed

Teacher Guide with Answer Sheets

Overview of Problem Set D

Estimated class time: 1 to 2 hours

In this Problem Set, students will be introduced to the effects of speed changes on airplane spacing.

In the previous *LineUp With Math*™ Problem Sets, students used route changes to resolve spacing conflicts. In this Problem Set, students begin to explore the effects of speed differences on airplane spacing. This will prepare them for subsequent Problem Sets where they resolve spacing conflicts via speed changes.

Objectives

Students will:

- Learn that when a plane's speed is reduced, the difference in the plane's distance traveled can be calculated by multiplying the difference in speed by the time traveled. (For a mathematical derivation of this relationship, see Appendix II.)
- Learn how to relate a decrease in plane speed in knots to a decrease in distance traveled each minute.
- Learn the correspondence between a 60-knot reduction in plane speed and the resulting decrease in distance traveled (in 1 minute) over a 10 Nmi interval on a jet route. (Students will encounter these particular speed reductions and multiples of 10 Nmiles in subsequent workbooks.)

Prerequisites

Before attempting the current Problem Set, it is *strongly* recommended that students complete Problem Set A that provides essential air traffic control vocabulary, units, and representations.

Materials

- Student Workbook D (print-based)

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The materials are available on the *LineUp With Math*™ website:

http://www.smartskies.nasa.gov/lineup

Student Workbook

It is recommended that you have a copy of Workbook D open while you read these notes.

The Workbook consists of four worksheets.

For a complete set of answers to each worksheet, see Appendix I of this document.

For each worksheet, the key points are briefly described as follows.

Worksheet: *Introduction to Travel at Different Speeds*

- Students will use two number lines to plot the positions (in one-minute intervals) of two students walking at different rates. Since the difference in rates is 1 step per minute, the slower student will fall behind 1 step each minute. So over a period of 5 minutes, for example, the slower student will fall behind 5 steps.
- When the difference in rates is 2 steps per minute, the slower student will fall behind 2 steps each minute. So over a period of 5 minutes, for example, the slower student will fall behind 10 steps.

Worksheet: *Change Knots to Nautical Miles per Minute*

- In the previous Workbooks, students worked with plane speeds in knots (nautical miles per hour). However, controllers need to make decisions in minutes. So students learn how to change knots to nautical miles per minute.
- In particular, students practice relating a 60-knot (nautical miles per hour) speed decrease to its equivalent 1 nautical mile per minute speed decrease.

Worksheet: *Plot Distances for Different Plane Speeds*

- This worksheet revisits the same speed relationships introduced in the previous worksheet that featured two planes flying at different speeds. The plane speeds differed by 60 knots. That corresponds to a speed difference of 1 nautical mile per minute. In the current worksheet, rather than plot plane positions on number lines, the students plot plane positions on the sector diagram.
- The students generalize the relationship between speed difference and distance traveled in the same time to examine a speed difference of 2 nautical miles per minute. They apply proportional reasoning to answer several questions.

Answer Sheets

Answer sheets for each worksheet in Student Workbook D can be found in Appendix I of this document.

For a mathematical derivation of the relationship between the difference in plane speed and the difference in distance traveled, see Appendix II of this document.

LineUp With Math™

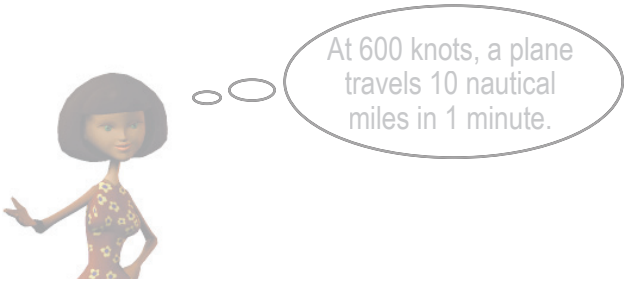
Math-Based Decisions in Air Traffic Control

Student Workbook D

Appendix I

- Understanding the Effects of Differences in Speed
  - Plot distances traveled at different speeds
  - Compare nautical miles per minute

Workbook Answers



Investigator: _____

An Airspace Systems Program Product

Introduction to Travel at Different Speeds

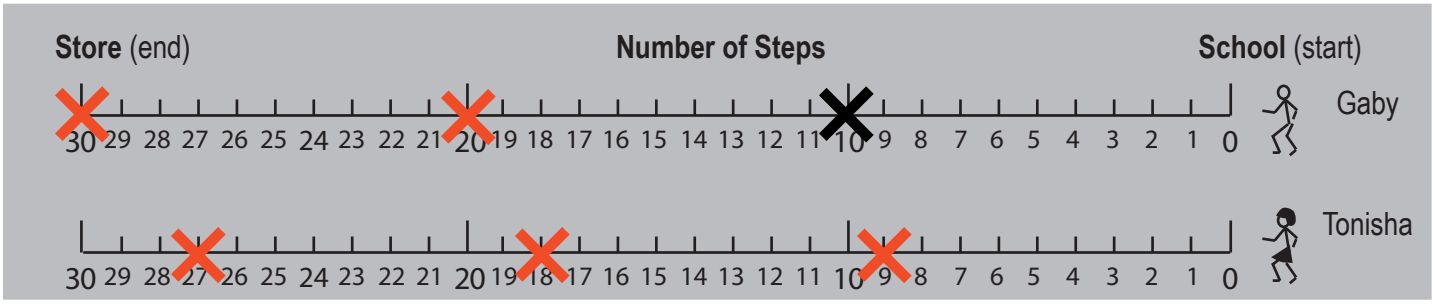
Investigator: _____

- Gaby and Tonisha are walking from school to a store. (Neither has a "headstart.")
- Each walks at a different speed (steps/minute) as shown in the speed table.
- Gaby and Tonisha each take the same size steps.

Speed Table

Name	Speed
Gaby	10 Steps/minute
Tonisha	9 Steps/minute

1 The number of steps that Gaby takes in 1 minute is: **10** steps Tonisha: **9** steps



- On Gaby's line, an ✕ is shown where he will be in 1 minute.

3 On Tonisha's line, put an ✕ where she will be in 1 minute.

4 How many steps is Tonisha behind Gaby after 1 minute? **1** steps

4 Mark Gaby's position and Tonisha's position after 2 minutes.

5 How many steps is Tonisha behind Gaby after 2 minutes? **2** steps

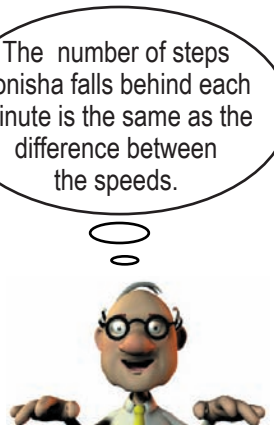
6 Mark Gaby's position and Tonisha's position after 3 minutes.

7 How many steps is Tonisha behind Gaby after 3 minutes? **3** steps

8 How many steps does Tonisha fall behind Gaby **each** minute? **1** steps per minute

9 How many steps would Tonisha fall behind in 5 minutes? **5** steps

10 If Tonisha takes 8 steps per minute, how many steps would she fall behind Gaby in 5 minutes? **10** steps



Speed difference is now 2 steps/min
2 steps/min • 5 min = 10 steps

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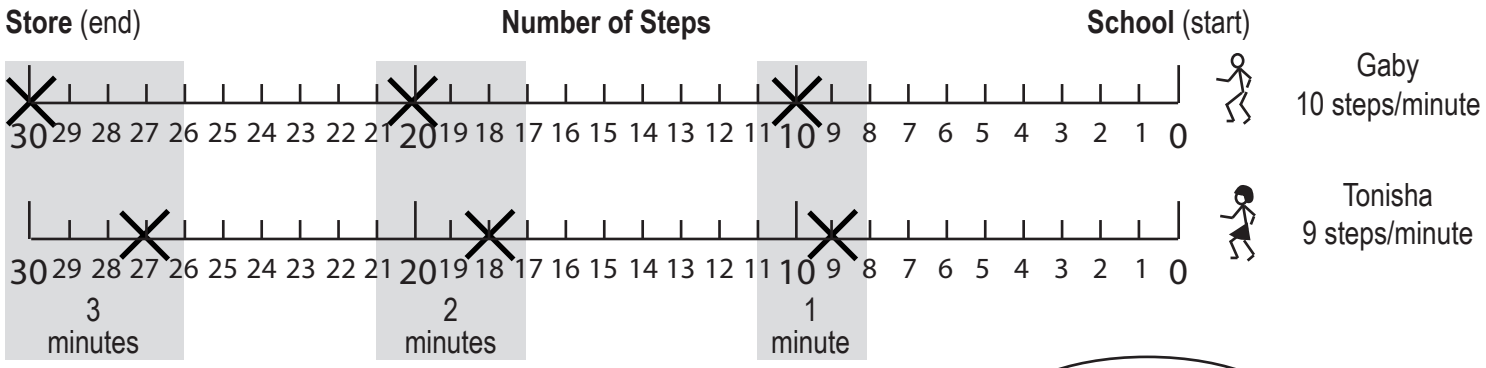
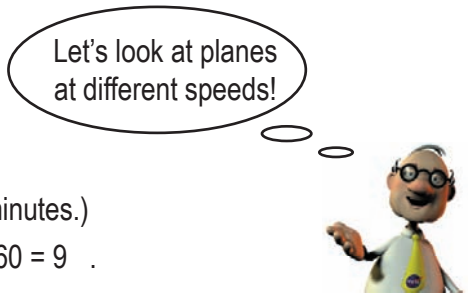
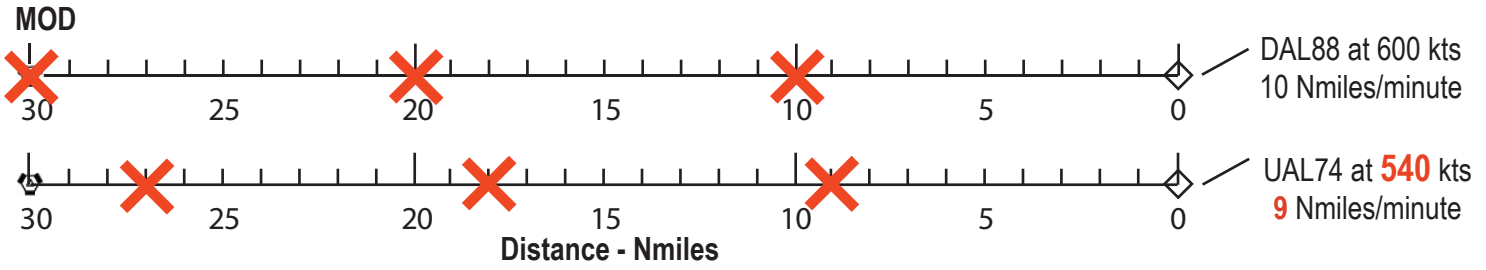
Introduction to Travel at Different Speeds (continued)

Investigator: _____



- DAL88 and UAL74 are each **30** Nmiles from MOD.
- DAL88 is traveling at 600 kts. That's 10 Nmiles per minute.
  (In 1 minute, the plane travels 1/60th the distance it travels in 60 minutes.)
- UAL74 is traveling at 540 kts. That's 9 Nmiles per minute. 540 • 1/60 = 9 .





6 On the DAL88 line, put an X through the number of miles it will travel in 1, 2, and 3 minutes.

7 On the UAL74 line, put an X through the number of miles it will travel in 1, 2, and 3 minutes.

6 How many miles is UAL74 behind DAL88 after:

1 minute: **1** Nmiles 2 minutes: **2** Nmiles 3 minutes: **3** Nmiles

6 How many fewer nautical miles will DAL88 travel in **each** minute? **1** Nmiles per minute

7 When DAL88 has traveled 30 Nmiles to MOD, how many Nmiles behind is UAL74? **3** Nmiles

- **At 600 kts, a 60-knot speed drop causes a 1 Nmile distance drop *every 10 Nmiles*.**

End of Worksheet

Change Knots to Nautical Miles per Minute

Investigator: _____

Recall: 1 Knot = 1 Nautical Mile per Hour
1 Hour = 60 Minutes

- Since planes fly so fast, air traffic controllers need to make decisions in minutes.
- To do this they need to know how many nautical miles a plane will travel in 1, 2 and 3 minutes.

1 To change from nautical miles per hour (knots) to nautical miles per minute, divide by **60**.

Speed in knots (Nmiles/hour)	To change Knots to Nmiles per minute, divide by 60	Speed in Nmiles/minute
600 kts	600 ÷ 60 = 10	10 Nmiles/minute
540 kts	540 ÷ 60 = 9	9 Nmiles/minute
480 kts	480 ÷ 60	= 8 Nmiles/minute

In 1 minute, a plane travels 1/60th the distance it travels in 60 minutes.

2 In the table below, fill in the total distance a plane travels in the times shown for each speed.

	1 minute	2 minutes	3 minutes
600 kts	10 Nmiles	20 Nmiles	30 Nmiles
540 kts	9 Nmiles	18 Nmiles	27 Nmiles

3 With a 60-knot speed *reduction*, how much *less* distance does the plane travel in the times below?

Speed Reduction	1 minute	2 minutes	3 minutes
60 kts	1 Nmile less	2 Nmiles less	3 Nmiles less

4 If a plane slows its speed by 60 knots, how many nautical miles *less* will it travel each minute? 1 Nmiles less

- Controllers reduce speed in 60 knot steps because it is easy to remember this rule:

For every 60-knot drop in speed, there is a 1-Nmile drop in distance each minute.

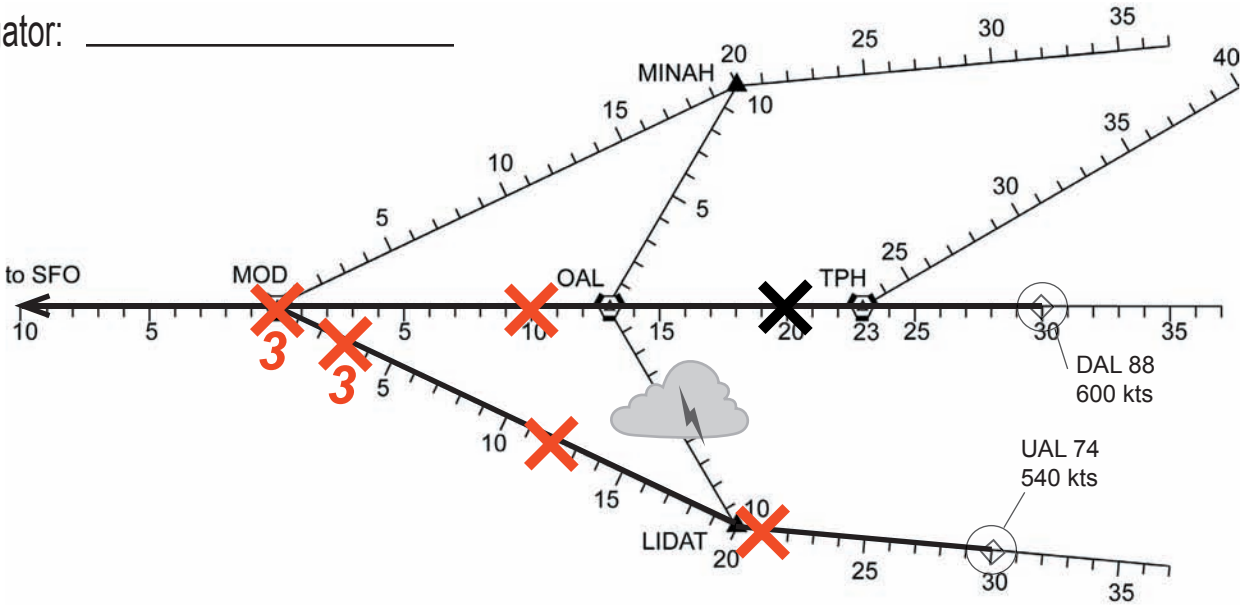
5 A controller reduces a plane's speed from 600 kts to 540 kts. How many Nmiles *less* will the plane travel in 5 minutes? 5 Nmiles less

1 Nmi/min • 5 mins = 5 Nmi

End of Worksheet

Plot Distances for Different Plane Speeds

Investigator: _____



This table shows plane speeds.

Call Sign	Speed Knots	Speed Nmi per Minute
DAL88	600	10
UAL74	540	9

1. How many Nmiles does each plane travel in 1 minute? DAL88 **10** Nmi UAL74 **9** Nmi

2. For **each** plane, use an ✕ to plot its position at 1, 2, and 3 minutes. Put a 3 near each plane's 3-minute mark: ✕$_3$

3. How many Nmiles does UAL74 fall behind DAL88 each minute? **1** Nmiles per minute

4. Using the speed table, the difference in plane speeds **in Nmiles per minute** is: **1** Nmiles per **minute**

5. The number of Nmiles that UAL74 falls behind each minute is the
   ☒ same as ☐ different than the difference between plane speeds in Nmiles per minute.

6. How far will UAL74 fall behind in 3 minutes? **3** Nmiles

7. Suppose the difference in speed is 2 Nmi/minutes.
   - How far would UAL74 fall behind in 3 minutes? **6** Nmiles
     2 Nmi/min difference • 3 min = 6 min
   - How many minutes will it take UAL74 to fall 8 Nmi behind? **4** Minutes
     8 Nmi ÷ 2 Nmi/min = 4 min

End of Worksheet

Appendix II

Workbook D – Difference Method:

Two planes are traveling at the same speed to MOD. When one plane's speed is reduced, the difference between the distances traveled by the planes in a given amount of time can be calculated by multiplying the difference in plane speeds by the time traveled.

To derive this relationship between the difference in speeds and the difference in distances traveled, we use the formula

$$\text{distance} = \text{rate} \bullet \text{time}.$$

Let d_1, r_1, and t be the original distance, speed, and time for each plane. Then

$$d_1 = r_1 \bullet t$$

Let d_2 and r_2 be the reduced distance and speed, respectively for one plane. The plane whose speed is **not** reduced will cover the distance to MOD in the original amount of time, t. We want to know where the plane with the reduced speed will be at this time t. So we again use t to represent time. We have

$$d_2 = r_2 \bullet t$$

The difference in the distances covered at the reduced speed is $d_1 - d_2$.
We have

$$d_1 - d_2 = r_1 \bullet t - r_2 \bullet t$$

$$= (r_1 - r_2) \bullet t$$

So,

$$d_1 - d_2 = (r_1 - r_2) \bullet t$$

Thus, the difference in distances traveled is equal to the difference in speeds times the original time.